Worksheet for Sections 8.7 and 9.1

1. Explain why, and where, the following integrals are improper.

   a. $\displaystyle\int_0^\pi \frac{\sin x}{\sqrt{1+\cos x}}\,dx$ b. $\displaystyle\int_0^{\pi/2} \sec^2 \theta\,d\theta$ c. $\displaystyle\int_1^\infty \frac{1}{x(x+1)}\,dx$ d. $\displaystyle\int_1^\infty \frac{1}{t^2-2t+1}\,dt$

2. Suppose that f is a differentiable function with the following graph:

   Determine which of the following integrals converge, and evaluate any that does converge.

   a. $\displaystyle\int_{-\infty}^0 f'(x)\,dx$ b. $\displaystyle\int_0^\infty f'(x)\,dx$ c. $\displaystyle\int_{-\infty}^\infty f'(x)\,dx$

3. (a) From (1) on p. 569, the Taylor polynomial p_5 for a function f is given by

   $$p_5(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \frac{f^{(5)}(0)}{5!}x^5$$

   Show that $p_5^{(5)}(0) = f^{(5)}(0)$.

   (b) Let a, b, and c be nonzero constants, and let $f(x) = ax^2 + bx + c$ for all x. Write down formulas for the polynomials p_2 and p_4, and explain how you arrived at them.

4. Let $f(x) = \sin x$. We want to find $p_8(x)$.

   (a) Find $f'(x)$ and $f'(0)$, $f''(x)$ and $f''(0)$, ..., $f^{(8)}(x)$ and $f^{(8)}(0)$.

   (b) Using (a) and (1) on p. 569, write down $p_0(x), p_1(x), \ldots, p_8(x)$.

   (c) What pattern do you see in your answer to (b)?

   (d) On your graphing calculator (or computer), plot together the graphs of $p_0(x), p_1(x), p_3(x)$ and $p_8(x)$, along with the graph of the sine function f, and describe the relationship between the graphs of these 4 Taylor polynomials and graph of f

   (e) Using the result of (c), write down the term with highest degree of x in each of the following:

   (i) $p_{34}(x)$ (ii) $p_{51}(x)$ (iii) $p_{99}(x)$